THE INFLUENCE OF DIFFERENT EMBEDMENTS ON DYNAMIC RESPONSE OF FOUNDATION AND ITS SURROUNDING GROUND

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ABSTRACT

The present paper examines the dynamic response of foundation and its surrounding ground for different embedments. A simplified mechanical model of coupled horizontal and rocking vibration of embedded footing is proposed, considering the additional moment caused by the horizontal reaction of surrounding soil. The result of this paper is compared with those given by M. Novak and R. V. Whitman, also with the data obtained in the vibration measurement of buried foundation. In addition, the dynamic response of the surrounding ground of the foundation with various buried depth was measured also.

INTRODUCTION

The dynamic response of foundation can be highly affected by the embedment. Up to now, most studies of embedded foundation concern vertical vibration. Dynamic response of coupled horizontal and rocking vibration of buried foundation has not yet been further studied perfectly and scarce test data are obtained. The present paper examines the coupled horizontal and rocking vibration of embedded footing basically. Baranov, V. A. derived the relations between the vibration of foundation and the side reaction of horizontal soil layer based on the elastic half-space theory (5). Accordingly, M. Novak investigated the vibration of embedded foundation and gave the solution of coupled horizontal and rocking vibration of embedded foundation (1), (6). But there are six parameters in his displacement equations. Besides horizontal spring constant K_x, damping coefficient C_x and rocking spring constant K_φ, damping coefficient C_φ, there are also coupled spring constant $K_{x\varphi}$ and coupled damping $C_{x\varphi}$. Based on the inverse calculation of dynamic response of coupled vibration, only four parameters can be obtained. Then, it is impossible to invert dynamic response to obtain vibration parameters by means of Noval's method and to make use of the test result of those footings resting on the surface of ground to modify the parameters. Novak's method is not convenient for application. In order to apply in practice conveniently, R. V. Whitman simplified the buried footing as the footing resting on the surface of elastic half-space by means of the method of modifying parameters (4). In fact, the side reaction of horizontal soil layers acts upon the side of foundation for buried foundations; and the horizontal reaction acts upon the bottom of foundation for footings resting on the surface of ground. The additional moment important for deep embedment is neglected in Whitman's. The present paper considers the additional moment and the result is more simple than Novak's.

THEORETICAL ANALYSIS

The soil beneath a base is assumed as the elastic half-space, the soil around the footing is assumed as total of independent horizontal elastic layers.

Dividing the spring constant and damping of a buried footing by the spring constant and damping of the footing resting on the surface of elastic half-space separately, the approximate expressions for effect of embedment can be obtained:

	K/K_0	D/D_0
Vertical	$n_z=1+0.6(1-\nu)\delta$	$\alpha_z=\dfrac{1+1.9(1-\nu)\delta}{\sqrt{n_z}}$
Swaying	$n_x=1+0.55(2-\nu)\delta$	$\alpha_x=\dfrac{1+1.9(2-\nu)\delta}{\sqrt{n_x}}$
Rocking	$n_\varphi=1+1.2(1-\nu)\delta+0.2(2-\nu)\delta^3$	$\alpha_\varphi=\dfrac{1+0.7(1-\nu)\delta+0.6(2-\nu)\delta^3}{\sqrt{n_\varphi}}$

Where, K and D are spring constant and damping ratio for a buried footing; K_0 and D_0 are spring constant and damping ratio for the footing resting on the surface of elastic half-space; embedment ratio $\delta = l/r_0$, l = depth of embedment, r_0 = radius of footing; ν is Poisson's ratio.

The research of elastic half-space shows that the spring constant and damping change with the dimensionless frequency a_0 ($a_0 = \omega r_0/V_s$), where ω is excitation frequency, V_s is velocity of shear wave (2). But, as $0 \leq a \leq 1$, the dynamic response based on constant parameters is very consistent with that according to "exact solution".

The simplified mechanical model of a coupled horizontal and rocking vibration of buried footing is presented in Fig.1. The footing is assumed as a rigid cylinder, subjected to horizontal exciting force $F_x e^{i\omega t}$ to cause coupled horizontal and rocking vibration. The motion condition of the buried footing is shown in Fig.1 (a), it is shown in Fig.1 (b) that the buried footing is simplified to a footing resting on the surface of elastic half-space. Due to the consideration of the additional moment T (t) caused by the horizontal reaction $N_x(t)$ of round soil layers in Fig.1 (b), the motion conditions of footing are equivalent for Fig. 1(a)and Fig.1 (b). The equations of motion for the simplified mechanical model shown in Fig.1 (b) are:

$$\left.\begin{aligned} &M\ddot{x}_g + R_x(t) = F_x e^{i\omega t} \\ &I\ddot{\varphi} + R_\varphi(t) - R_x(t)\cdot h_0 = F_x\cdot h_1\cdot e^{i\omega t} - T(t) \end{aligned}\right\} \quad (1)$$

Where, M = mass of footing; I = mass moment of inertia about centre of base.

$$R_x(t) = C_x\dot{x}_b + K_x x_b \quad (2)$$

$$R_\varphi(t) = C_\varphi\dot{\varphi} + K_\varphi\varphi \quad (3)$$

From the approximate expressions for the effect of embedment, K_x, K_φ, C_x and C_φ can be expressed as follow:

$$K_x = \eta_x\cdot K_{x0}\,,\qquad C_x = \alpha_x\cdot C_{x0}$$

$$K_\varphi = \eta_\varphi\cdot K_{\varphi 0}\,,\qquad C_\varphi = \alpha_\varphi\cdot C_{\varphi 0}\,.$$

Where, K_{x0}, $K_{\varphi 0}$, C_{x0} and $C_{\varphi 0}$ are spring constant and damping for footing resting on the surface of elastic half-space.

The additional moment T (t) is:

$$\begin{aligned} T(t) &= N_x(t)\cdot l_0 \\ &= G_s\, l(S_{u1} + iS_{u2})\, x_b e^{i\omega t}\cdot\frac{l}{2} \\ &\quad + G_s\, l^3(S_{u1} + iS_{u2})\frac{1}{3}\varphi e^{i\omega t} \\ &= \frac{1}{2}G_s l^2 [x_g + (\tfrac{2}{3}l - h_0)\varphi](S_{u1} + iS_{u2})e^{i\omega t} \end{aligned} \quad (4)$$

Where, S_{u1} and S_{u2} are Baranov's frequency function for layer. As $0 \le a_0 \le 1.5$, S_{u1} and S_{u2} can be taken as constants (3). G_s = shear modulus of elastic layer (back fill).

$$x_b = x_g - h_0\varphi \quad (5)$$

Substituting Eq. (2)~(5) in (1), the equations of motion are:

$$\left.\begin{aligned} &M\ddot{x}_g + C_x(\dot{x}_g - h_0\dot{\varphi}) + K_x(x_g - h_0\varphi) = F_x e^{i\omega t} \\ &I\ddot{\varphi} - C_x(\dot{x}_g - h_0\dot{\varphi})h_0 - K_x(x_g - h_0\varphi)h_0 + (C_\varphi\dot{\varphi} + K_\varphi\varphi) \\ &\quad = \{F_x h_1 - \tfrac{1}{2}G_s l^2[x_g + (\tfrac{2}{3}l - h_0)\varphi](S_{u1} + iS_{u2})\}e^{i\omega t} \end{aligned}\right\} \quad (6)$$

Let:
$$\left.\begin{aligned} &x_g = x_0 e^{i\omega t} \\ &\varphi = \varphi_0 e^{i\omega t} \end{aligned}\right\} \quad (7)$$

Substituting Eq. (7) in (6) and let:

$$a_1 = \frac{K_x}{M} - \omega^2 + i\omega\frac{C_x}{M}$$

$$a_2 = -\frac{K_x h_0}{M} - i\omega\frac{C_x}{M}h_0$$

$$a_3 = -\frac{k_x h_0}{I} - i\omega\frac{C_x h_0}{I} + \frac{\frac{1}{2}G_s l^2(S_{u1}+iS_{u2})}{I}$$

$$a_4 = \frac{k_\varphi + k_x h_0^2}{I} - \omega^2 + i\omega\frac{C_\varphi + C_x h_0^2}{I} + \frac{\frac{1}{2}G_s l^2(S_{u1}+iS_{u2})(\frac{2}{3}l - h_0)}{I}$$

The solution is:

$$\left.\begin{aligned} X_0 &= \frac{\frac{F_x}{M}a_4 - \frac{F_x h_1}{I}a_2}{a_1 a_4 - a_2 a_3} \\ \varphi_0 &= \frac{\frac{F_x h_1}{I}a_1 - \frac{F_x}{M}a_3}{a_1 a_4 - a_2 a_3} \end{aligned}\right\} \qquad (8)$$

The absolute values $|X_0|$ and $|\varphi_0|$ are horizontal and rocking amplitudes separately.

The amplitude of swaying vibration on the top of footing is:

$$X = |X_0| + (h - h_0)|\varphi_0| \qquad (9)$$

EXPERIMENTS WITH EMBEDDED FOOTING

The soil 18 m deep under the surface is loess-like silty clay. In a depth of 5 m, the mechanical properties of soil are: mass density of soil beneath to footing base $\rho = 1.78 \times 10^{-6}$ kg-sec^2/cm^4 ; void ratio $e = 0.95$; $\nu = 0.3$; $V_s = 130\sim149$ m/sec; shear modulus of soil beneath to footing base G = 300~400 Kg/cm^2 (converting from V_s).

In the field test, the footing is a concrete block, its dimension is 150 cm × 150 cm × 150 cm, its weight is 8.44 t. The space between the footing and the adjacent soil is 100 cm in width and 150 cm in depth. The hole is filled up by three times, the soil filled in the hole is about 1/3 depth of the hole each time, the depth of the fill is $l = 0$, 45 cm, 105 cm and 145 cm separately. The fill is tamped, its mass density corresponds to $\rho_s = 0.8\,\rho$. When there is no fill in the hole, the footing can be considered to rest on the surface of ground, because the hole is wider and more shallow. The measurement of dynamic response of footing is done to include four conditions: $\delta = 0.0$, 0.532, 1.241 and 1.714.

COMPARISON OF THEORY WITH EXPERIMENTS

The value of G is estimated according to the following formula:

$$G = 326\,\frac{(2.97 - e)^2}{1 + e}\sqrt{\sigma_0} = 360 \text{ Kg/cm}^2$$

in which, the effective stress of "typical" point under the footing $\bar{\sigma}_0 = 0.28$ Kg/cm^2 .

The spring constant of soil is obtained from the inverse calculation of the measured dynamic response, and then the value of G is obtained from the inverse calculation of the spring constant again, G = 340 Kg/cm^2 .

The calculated value of G is close to the estimated one, and agrees with the value of G calculated from V_s . In order to compare the theoretical and measured resonant curves, in the following calculation, G will be taken as 340 Kg/cm^2.

For backfill, $G_s = (\frac{\rho_s}{\rho})^3 \cdot G$. The coefficients of approximate expressions for effect of embedded depth are reduced correspondently.

As $\delta = 0.0$, the vibration parameters of footing according to the formulas of lumped parameters are given in Table 1. But, among them, the damping in rocking vibration is given according to the following expression:

$$D_\varphi = (0.03 \sim 0.05) + 0.1/\sqrt{b_\varphi}(1 + b_\varphi/4) \qquad (10)$$

in which, mass ratio $b_\varphi = I/\rho r_0^5$

The vibration parameters of footing for different embedded depth are given in Table 2.

In the calculation of this paper, functions of a_0 for elastic half-space $C_{j1,2}$ and Baranov's frequency functions for layer $S_{j1,2}$ are constants:

C_{u1}	C_{u2}	$C_{\varphi 1}$	$C_{\varphi 2}$	S_{u1}	S_{u2}	$S_{\varphi 1}$	$S_{\varphi 2}$
4.71	2.96	3.81	0.39	4.10	10.6	2.50	1.80

The theoretical and measured response curves of foundation for different embedments are shown in Fig.2 and 3. The theoretical ones are obtained by three kinds of method: Novak's,Whitman's and the method proposed by this paper.

Obviously, as $\delta = 0.0$, the three curves are well consistent and coincide as one curve. This curve is well fit with the measured data, the measured curve for rocking vibration is somewhat higher than theoretical only after 30 Hz . For other embedded depth, the measured data are approximately agree with the theoretical ones given by this paper's. When δ is smaller, Whitman's curve approaches to the measured data; but as δ increases Whitman's curves leave from the measured data gradually. Novak's curves also approach to the measured data. If the value of G is not calculated by the method of this paper correctly, Novak's curves would not approach to that of the measured data.

While amplitude decreases with the increase of buried depth, the tendency is shown in Fig.4. According to the methods given by this paper and Novak amplitude attenuates more faster with buried depth when compared that given by Whitman's method.

The largest horizontal resonant amplitudes for different embedments are given in Table 3. Deviation of Whitman's method increases with buried depth, which are 1.6% , 1.66, 3.74 and 5.32 times successively. It shows that the additional moment is very important for great depth.

The deviation for the method of this paper is very little. In the displacement expressions of this paper, there are only four vibration parameters which can be obtained to calculate inversely dynamic response of footing by means of the method of phase difference. This method is more simple and convenient for application.

DYNAMIC RESPONSE FOR SURROUNDING GROUND

The arrangement of pick-ups in the test is shown in Fig.5. Vertical and horizontal steady excitation has been done separately.

For the ground motion near the source of excitation, it is not enough to consider the R - wave only, the influence of P - wave must be considered. There is no accurate solution for the wave field shown in Fig.5. But, the measured data of ground motion approach those given by the following expression:

$$A_r = A_0 \beta \sqrt{\frac{r_0}{r}\left[1 - \xi_d \left(1 - \frac{r_0}{r}\right)\right]}\; e^{-f\alpha_0 (r - r_0)} \qquad (11)$$

where, A_r =amplitude of surface of ground to distance r from centre of footing; A_0 = amplitude of footing; β= coefficient for influence of loading, for nature surface of ground β =1; f = excitation frequency; ξ_d = coefficient of influence of P - wave, it depends upon r_0 ; α_0 = coefficient for absorption of energy of soil. In this paper, ξ_d = 0.7, α_0= 2.0×10^{-3} .

The wave propagation along the surface of ground can be seen in Fig.6 to 9. The origin of coordinate locates on the side of footing, the abscissa represents distance from the side of footing. Two phenomena are observed: on one hand, the space (hole) around the footing obviously has the effect of vibration isolation, and this effect is more obvious for deep hole than that for shallow one. On the other hand, the vibration of footing and its surrounding ground is strong for deep hole than that for shallow one. Because the hole becomes deeper, the embedment of footing would decrease, the vibration of footing and its surrounding ground would be more intense. Accordingly, an important conclusion can be obtained that it is unfavorable to install a void or space around the vibrating foundation. Compaction of the backfill around the vibrating foundation is necessary to decrease the dynamic response of the footing and its surrounding ground.

CONCLUSIONS

The method proposed by this paper and experiments agree qualitatively in the decrease in resonant amplitudes and increase in resonant frequencies with embedment depth. The simplified mechanical model is rational and the method to determine the parameters is rellable.

For the footing subjected to excitation, it is better to compact the backfill than to install a void around the footing to decrease the vibration of foundation and its surrounding ground.

If vibration of a footing continues for long time, it will be possible to produce a void around the footing for cohesion soil. The backfill must be compacted and made more dense. Perhaps, it is effective to fill the expansive material around the footing to strengthen the effect of embedment.

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Table 1. δ =0.0, Vibration Parameters of Footing

Model	Spring constant K	Mass ratio B	Damping ratio D	Additional mass
Swaying	1.353×10^{5} Kg/cm	1.57	0.22	0.0605 M
Rocking	0.7843×10^{9} Kg-cm	1.02	0.05	0.235 I

Table 2. Vibration Parameters of Footing for Defferent Embedded Depth

δ	Swaying		Rocking	
	K_x (Kg/cm)	D_x	K_φ (Kg-cm)	D_φ
0.532	1.695×10^{5}	0.3689	0.9796×10^{9}	0.0542
1.241	2.153×10^{5}	0.5313	1.448×10^{9}	0.0842
1.714	2.457×10^{5}	0.6246	2.020×10^{9}	0.1246

Table 3. Largest Horizontal Resonant Amplitudes on the Top of Footing for Defferent Embedded Depth

δ		0.0	0.532	1.241	1.714
Measured data (μ)		127.0	44.1	9.3	3.7
Theoretical value (μ)	this paper's	125.0	46.5	9.6	4.3
	Whitman's	125.0	73.0	34.8	19.7
	Novak's	125.0	39.5	10.5	4.6

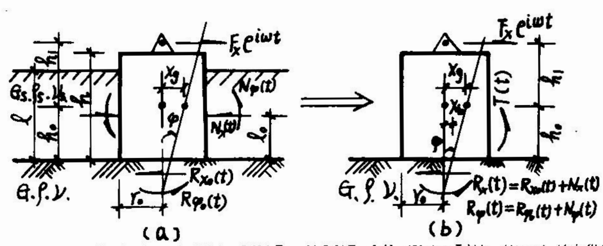

FIG.1 SIMPLIFIED MECHANICAL MODEL OF COUPLED HORIZONTAL AND ROCKING VIBRATION OF EMBEDDED FOUNDATION

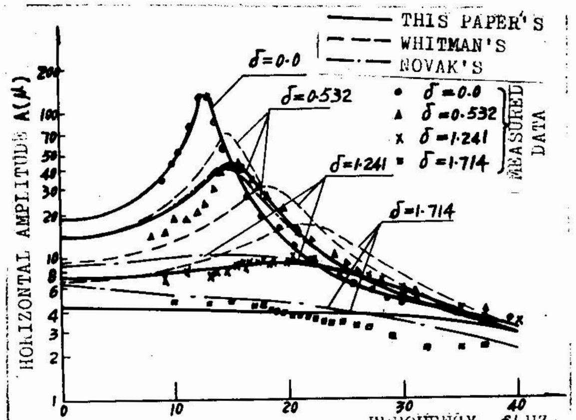

FIG.2 THEORETICAL AND MEASURED DYNAMIC RESPONSE FOR HORIZONTAL VIBRATION

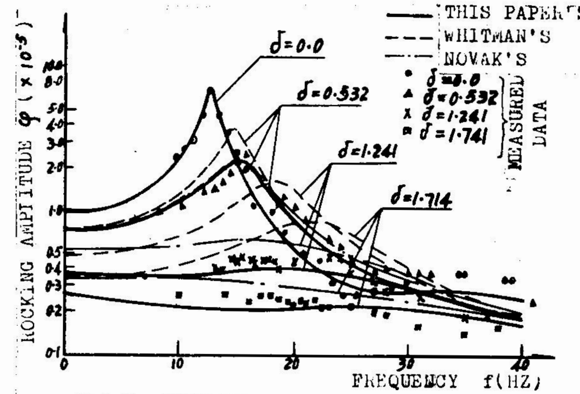

FIG.3 THEORETICAL AND MEASURED DYNAMIC RESPONSE FOR ROCKING VIBRATION

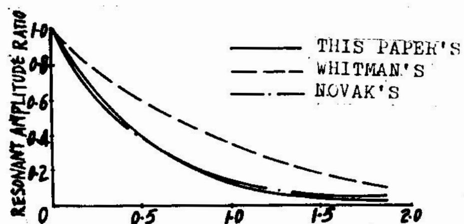

FIG.4 THEORETICAL RESONANT AMPLITUDE REDUCTION FOR DIFFERENT METHODS

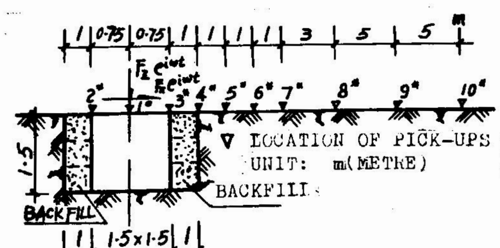

FIG.5 THE ARRANGEMENT OF PICK-UPS

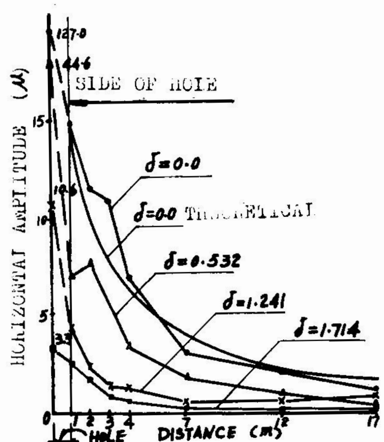

FIG.6 HORIZONTAL RESPONSE OF SURROUNDING GROUND BY HORIZONTAL EXCITATION

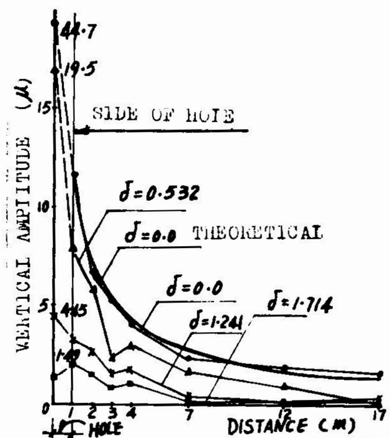

FIG.7 VERTICAL RESPONSE OF SURROUNDING GROUND BY HORIZONTAL EXCITATION

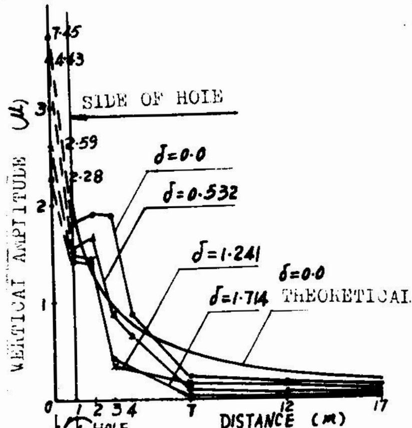

FIG.8 VERTICAL RESPONSE OF SURROUNDING GROUND BY VERTICAL EXCITATION

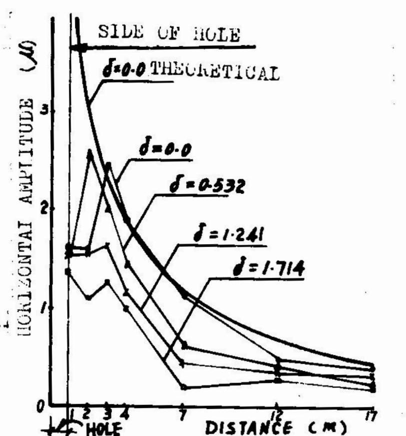

FIG.9 HORIZONTAL RESPONSE OF SURROUNDING GROUND BY VERTICAL EXCITATION